Judges Commentary: Traffic Circles

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1 Overview of the Problem

Students who decided to explore the "A" problem in this year's Mathematical Contest in Modeling examined ways to control the flow of vehicles moving through a traffic circle. Some of the methods that are available to control the flow of traffic were given in the problem statement. The methods given include the use of yield signs, stop signs, and traffic lights.

In this overview, I provide some observations about the judging of the entries in this year's competition. These observations make use of input from the observations of other judges as well as my own.

First, a broad overview of the criteria that was developed from our initial reading of the problem is presented. In the section that follows, section 2, a brief overview of the problem statement is explored. In section 3, an overview of the judging itself is given. In section 4 a list of some of the common approaches adopted by the students is given. Finally, in section 5 a list of some of the common themes and more detailed points that emerged as the judging proceeded is given.

2 Traffic Circles

The focus on the "A" problem is on controlling the movement of vehicles through a traffic circle. A number of explicit controls that are available are explicitly given in the problem statement. The student's who submitted papers for this problem mainly focused on the given controls and very few examined other types of controls.

Prior to the evaluation of the papers the problem statement was sent to the judges. The first thing we did in our initial meeting was to share our interpretations of the question and share what we felt were the most important parts of the question. This year a consensus quickly emerged about the essential aspects which was somewhat unusual.

Part of the reason for our quick convergence is that the problem statement explicitly asks that the students do two things. First, the students were asked to find a way to control the flow of traffic in an optimal way. Second, the students were asked to write a summary of their findings. These two aspects are explored individually in the subsections that follow.

2.1 The Goal

The goal for this problem is to find a way to move vehicles through a traffic circle in an optimal way. This was clearly and succinctly given in the second paragraph of the problem statement :

> "The goal of this problem is to use a model to determine how best control traffic flow in, around, and out of a circle."

What is not clear is what “best” might mean. This key aspect was left open for the students to decide what it means. Because it was left open for interpretation the students were required to make it clear in their report how they themselves interpreted this part of the problem:

> "State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice."

The judges paid close attention to this part of the modeling effort. We expected that the students clearly describe the objectives, and we expected that the subsequent evaluation of the model be consistent with the stated objectives. This can be a difficult thing for the student groups to achieve given the dynamic of writing as a team, the nature of how the modeling efforts evolve as the problem is explored, and the intense time pressure. Teams that managed to maintain some level of consistency stood out from the entries and tended to elicit a more positive response from the judges.

2.2 Technical Summary

An essential requirement was to write a technical summary. The requirements for the technical summary were explicitly given in the problem statement. This was a difficult aspect to the problem. In a short amount of space the teams were expected to provide a broad set of guidelines for a traffic engineer.

The traffic engineer would expect to read the summary and have a strong sense of what methods to implement to control the traffic in a variety of circumstances. Student teams included a number of different factors to include in their summary. Some of the items that were included were the radius or geometry of the circle, the rate of flow of traffic coming into the circle, the density of traffic coming into the circle, and other factors. Very few teams considered the potential capacity of traffic leaving the circle and assumed that the incoming traffic was a primary limiting factor.

In addition to the details of how to use the model, the traffic engineer is expected to have a broad understanding of the conditions for which the model is applicable. Prior to implementing the controls and methods developed by the student teams the traffic engineer would likely want to know the conditions that the model is applicable. This implies that the engineer should be able to read the summary and have a basic understanding of how the model was developed and an understanding of the potential pitfalls.

This part of the problem was a difficult hurdle for the teams. A limited amount of space, two pages, was available for the students to achieve this aspect of the problem, and they had a diverse amount of information to convey in that space. The student teams that managed to convey a sense of the basic models, the underlying assumptions, and the limitations of their models tended to make a stronger impression.

3 Grading Process

Before discussing the different approaches to the problem a brief overview of the evaluation process is given. The papers are evaluated in three stages. There is an initial round which focuses on which papers to remove from the pool. The second, or screening round, focuses on which papers meet the minimal requirements for an advanced score. In the final round the judges focus on which papers meet the highest standards.

3.1 Initial Grading

The initial round is designed to remove the papers from the pool that are not likely to meet the standards in the second round. This round is conducted by a dedicated group who focuses on the papers in this initial round. Each paper is read by at least two people. Papers that are consistently scored with low marks are not passed on to the next round. Papers with mixed reviews may be read by more people. In the case where the initial reviewers are not sure the reviewers try to err on the side of caution and pass the paper on to the next round.

It is absolutely essential that a paper be well written and have a clear, concise summary to make it past the initial round. Unfortunately, the judges in the initial round do not have a great deal of time to read a paper, and a paper which does not provide a clear overview including results and a synopsis of the techniques used will not make a strong impression on the judges. Just as important, the summary and the rest of the paper must be consistent. Differences between the summary and the following pages can be immediately apparent and do not make a positive impression of the paper.

3.2 Screening Round

The screening round follows the initial round of judging. This round is done by a second group of judges. Prior to the start of this round the judges are given a report from the judges who took part in the first round. The judges are each given a different set of papers with a wide range of reviews from the initial rounds as a way to give them an idea of the kinds of approaches adopted by the student teams.

As the judges examine papers in this round they try to decide if the paper meets the minimal requirements to do well in the rounds that follow. The number of times a paper is read in this round varies from year to year. Again the judges try to err on the side of caution, but as the rounds proceed the criteria for doing well is increasingly stringent.

In this round it is still important to have a strong summary, but the need for consistency for the whole paper becomes more important. The need for proper citations and strong grammar also becomes increasingly important. This year a large body of work is available for the students to gain insight and use. It was even more important this year to make use of proper citations and make it clear what work was done by the team and what work was found in their search of the literature.

3.3 Final Rounds

In the final rounds of judging the focus is on finding the best submission. At this point each paper is read multiple times, and more time is available for each reading. The judges are able to focus more on each individual step and focus on consistency across the whole paper. The papers that remain in these final stages must maintain high scores to move forward.

The longer that a paper remains in the final rounds of judging the more likely it will receive a higher overall rating. This particular year all of the papers that made the final rounds were given at least a meritorious rating. A total of four papers were given an outstanding rating this year for the "A" problem.

4 Approaches

Traffic engineers have given a great deal of attention to the flow of traffic in roundabouts. The work that has been done influenced many of the teams who worked on this problem. The majority of approaches that were adopted by the teams can be roughly broken down into two types, deterministic and stochastic. Here we examine each of these approaches separately.

4.1 Deterministic

The teams that adopted a deterministic approach tended to make greater use of partial differential equation models based on conservation laws. There are a variety of different conservation laws that have been derived to model traffic flow. Such models tend to focus on relatively simple traffic geometries and require considerable adaptations to model a traffic circle.

At first glance a conservation law approach for a traffic circle avoids some of the issues associated with boundary conditions because it is a periodic geometry. Unfortunately, the actual implementation magnifies this issue when accounting for the exits and entrances of the feeder roads. This aspect of the problem tended to require the majority of the modeling efforts.

The second difficulty with this approach is to find an approximation to the solution. This is an extremely difficult task. The equilibrium solution to the equations are piecewise constant functions, and the conversation law gives rise to shocks. Given the complex boundaries the method of characteristics is very complicated. At the same time numerical approximations can be a daunting task since the techniques must account for upwinding.

4.2 Stochastic

The majority of teams made use of a stochastic approach. In general these approaches made use of either queues or networks, and it was not uncommon for teams to make use of a hybrid model combining the two approaches. A typical paper included an overview of the model, some theoretical results for a simple situation, and provided results for a computational model.

Teams making use of this kind of approach were expected to make use of proper citations because of the wide body of work available. The judges also paid more attention to the consistency across the whole paper. Some teams submitted some wonderful work, but their summary, model, and results were not consistent. For these papers it was immediately clear that the teams tried a variety of approaches, but their paper was not updated to reflect the direction of their final efforts.

Another issue that emerged with papers who took a stochastic approach is the disconnect between the sections in their paper discussing the theory and the numerical simulations. The top rated papers tended to provide some theoretical results for simplistic geometries or simulations. The majority of these went on to include the results of numerical simulations for the more complicated cases. It was rare to find a paper in which the team discussed the use of their numerical simulation on the simplified situation as a way to provide a benchmark for the numerical model. The few teams that did provide a confirmation of the numerical model made an immediate positive impression.

The other issue is how to report the results of simulation in a coherent manner. This can be a daunting task. The development of the model requires a probabilistic approach. The analysis of the numerical trials requires a shift to a statistical approach. Unfortunately, very few teams were able to make this transition. The majority of teams simply reported means and maybe standard deviations. Only a very small number of teams reported results using boxplots or histograms, and even fewer made use of any qualitative statistical methods.

Finally, when designing the numerical trials few teams examined a range of values for the parameters in their models. This is an important aspect that the judges expect every year. The sensitivity of the model to assumptions is a vital concern to any modeling effort. Every year the judges discuss this aspect of the problem before, during, and after our reading of the papers. We expect to see what happens to the model for small changes in parameters or assumptions. The few teams that did examine this aspect immediately caught the judges attention and were more likely to be seen in a favorable light.

5 Common Themes

In the previous section some observations specific to this year's competition are given. Here some general observations that come up each year are given. These issues are aspects to a paper that the judges discuss almost every year.

5.1 Summary

The summary is an important part of the team's entry. It is the first thing that a judge will read. The summary is the first impression, and it imparts the expectations for the whole paper. In terms of this competition, it is absolutely vital that a paper have a complete and well written summary to make it past the initial rounds. It is also vital that the details in the summary be consistent with the rest of the paper especially in the later rounds.

Writing a one page summary of the team's efforts is a difficult task. The teams are expected to provide a brief overview of the problem. They are then expected to let the reader know their specific conclusions and recommendations. Finally, the teams are expected to provide the reader with an overview of the approach that they used.

It is difficult to include all three of these parts within the one page summary. Many teams find it tempting to include a large amount of background information or provide clever narratives motivating the problem. Unfortunately, such material in the summary can drastically reduce the amount of space available to discuss the team's results and discussion of the approach that they adopted.

5.2 Grammar, Punctuation, and Equations

The presentation of the team's model and results cannot be separated from the model itself. A team can have an ideal model including a complete analysis of the model. If the team is unable to share their results with a clear and concise discussion then their work is not relevant.

Teams that do not make use of proper grammar and punctuation are not likely to make it past the initial rounds of the competition. Teams are expected to know how to include equations in their writing and use proper punctuation. Advisers should not take it for granted that their students know how to do this. Unfortunately, many students do not get a chance to engage in formal writing in their course work, and this competition is their first experience in doing so.

A paper that is a simple narrative of the team's activities with equations poorly integrated into the text will not receive a high rating.

5.3 Proper Citations

Acknowledging the work of others is one of our most cherished principles. Unfortunately, too many students are only exposed to this part of their education in humanities and social science courses. Students often do not make this connection with respect to their mathematics writings. Advisers should not assume that their teams understand the proper use of citations in mathematical writing.

The growth of the web has accentuated the issue. Many student teams are comfortable exploring the resources available to them, and it is unusual to come across an entry with a unique approach. The different types of approaches can be easily categorized, and the judges quickly figure out the source for each approach.

5.4 Sensitivity and Stability

Every year the judges explicitly discuss the issue of sensitivity and stability. The few teams that make a concerted effort to explore this aspect of their model will almost always stand out, and this part of their efforts will be seen in a positive light. Exploration of the sensitivity of a model can be as simple as exploring what happens for a different ranges in a parameter to the use of more sophisticated methodologies such as an exploration of a sensitivity matrix.

Each year students are able to implement nontrivial numerical simulations. The students must make decisions about what numerical trials they should examine. It is extremely rare for students to first scale a problem as a way to decide the combination of parameters that are important with respect to deciding which numerical trials to use.

5.5 Figures and Tables

The integration of graphs and tables into a paper is a challenge for many students teams. It is not uncommon to see entries in which figures and tables are included but have no detailed discussion associated with them. The teams need to make the effort to integrate the figures and tables into their discussion.

Given the increased use of simulations and numerical results it is vital that the student teams find a way to weave their figures and tables into their presentation. The figures and tables are part of the story the students need to tell and are not something that stands apart from their discussion. The teams need to make sure to let their readers know the key aspects of their figures and tables and inform their readers how to look at the figures and tables.

5.6 Consistency Across the Paper

The teams have a limited time to understand the problem, derive a mathematical description of the problem, perform the requisite analysis of their model, and then come back and interpret their work with respect to the original physical system. Over the course of the weekend they must make many decisions and often explore different approaches that they must later abandon. The time constraints make it extremely difficult to complete a paper in which the wide array of assumptions and analyses are consistent across the whole paper.

For example, it is very common for student teams to offer a perfunctory bulleted list of their assumptions. On closer inspection, however, they often include insights sprinkled throughout their analysis that may contradict what is found in their bulleted list. The same thing is true of the team's determination of the strengths and weaknesses of their analysis.

A team that produces an entry that has an approach that is consistent with its summary and the assumptions are consistent throughout the paper greatly improves the chances that the judges will have a positive reaction when we read it.

6 Conclusions

The judging for the MCM includes several different stages. A team's submission must satisfy a wide array of criteria to be successful and proceed through each stage. The presentation and grammar is a vital aspect of a submission. The team's results are given through the filter of the team's writing.

A team's writing and grammar must be excellent for a submission to receive the highest ratings. The team must also provide a strong analysis. The team's efforts only last four days, and the judges do not expect extensive and sophisticated models. A careful analysis of the resulting model is required, though.

Each year the expectations are different, but there are a few constants. For example, a clear discussion of the basic assumptions with some justification and discussion of the implications is necessary. Additionally, a focused discussion on stability and sensitivity is necessary for an entry to receive a higher score.

In this year's competition the use of simulation was a part of the majority of entries. Incorporating an analysis of simulations is a difficult task, and the top entries did a remarkable job of integrating the development and analysis of their model with the discussion of the results of their numerical trials.

Teams that were able to tie together the theoretical analysis of their model along with their numerical trials received immediate positive attention. The best were able to develop multiple models of different complexity and then verify their numerical models with the theoretical results of the simpler models.