

The 2023 HiMCM

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Congratulations to our 2023 Outstanding team winners and all teams participating in our twenty-sixth International High School Mathematical Contest in Modeling (HiMCM)[®]. We are excited to again join the National Council of Teachers of Mathematics (NCTM) to designate two of our teams as NCTM Award winners. The HiMCM continues to be an amazing and rewarding experience for students, advisors, schools, and judges across the globe. A total of 967 teams, with up to 4 students each, representing 417 schools and 18 countries/ regions, competed this year.

Outstanding Teams

- •13643 Shanghai World Foreign Language Academy, Shanghai, China
- •13694 St. George's School, BC, Canada
- •13719 Nanjing Foreign Language School, Jiangsu, China
- •13904 Shanghai Starriver Bilingual School, Minghang, China
- •14076 Shanghai Pinghe School, Shanghai, China
- •14087 BASIS International School Guangzhou, Guangdong, China
- •14140 Shanghai Pinghe School, Shanghai, China
- •14323 Durham Academy, NC, USA (NCTM Award)
- •14632 Mass Academy of Math & Science, MA, USA (NCTM Award)

The 2023 Contest

Once again, the set of participating teams in 2023 was a joy for the judges to read with some truly impressive papers. COMAP seeks to continue the tradition of creating challenging realworld problems that are interesting for the students. As in the past, students can choose from two problems for the HiMCM. This year's problems challenged teams to investigate population growth and pollination capacity of honeybees in Problem A: Dandelions: Friend? Foe? Both? Neither? and Problem B: Charging Ahead with E-buses. The judges continue to be extremely impressed with the student's drive and mathematical abilities to compete in this modeling contest. We understand the challenges of time and resources put on students and would like to thank all participants and advisors who competed in this year's HiMCM contest.

Overview

COMAP has offered international modeling contests for over 40 years, with HiMCM celebrating its 26th contest in 2023. As an increasing number of schools engage their students in mathematical modeling, we see continued increasing participation in COMAP's modeling contests. Starting with 115 students in the first year of the HiMCM, over the course of 26 contests we have had 48,147 students apply their mathematical knowledge and skills as they modeled challenging problems in the HiMCM.

COMAP would like to recognize the value and importance of the student team advisors and teachers. These are the true champions who see the value of their students participating in a math modeling contest. These are the educators and mentors who encourage their students to go beyond the standard curriculum. We see many team advisors year after year involved with one or more teams from their educational institutions. COMAP is truly thankful for these individuals.

The 2023 contest had 967 submissions. Of the 967 submissions, 506 completed Problem A: Dandelions: Friend? Foe? Both? Neither, and 461 completed Problem B: Charging Ahead with E-buses. Table 1 shows the judging results of the 2023 HiMCM. We accept partial solutions and encourage all registered teams to submit a solution paper to experience the learning impact and satisfaction of fully participating in this challenging contest.

In total, 3,603 students participated in the 2023 HiMCM. Schools from a wide range of countries/regions competed, including teams from Australia, Canada, China, Germany, Hong Kong (SAR), India, Macau (SAR), Malaysia, New Zealand, Philippines, Singapore, South Korea, Taiwan China, Thailand, the United Arab Emirates, the United Kingdom, the United States of America, and Vietnam.

Rules

One important rule is that students may only use the members of their team along with inanimate (non-living) sources to complete the contest problem. Students may not use any chat rooms, electronic communication, or social media sources. Each year we have some teams that do not understand this rule. To be clear, contacting an expert in a



Problem	Outstanding	%	Finalist	%	Meritorious	%	Honorable Mention	%	Successful Participant	%	Unsuccessful	%	Disqualified	%	Total
А	5	1%	28	6%	70	14%	134	26%	262	51%	2	<1%	5	1%	506
В	4	1%	26	5%	69	15%	122	27%	239	52%	1	<1%	0	0%	461
Total	9	1%	54	6%	139	14%	256	26%	501	52%	3	<1%	5	1%	967

Table 1: 2023 HiMCM Judging Results



Figure 1: Total HiMCM Student Participants 1999 – 2023

field or an author of one of the referenced sources is a violation of this rule. Gathering data from people outside of your team using an interview or a survey or a questionnaire is a violation of this rule. Using solutions shared electronically by other teams or by organizations is a violation of this rule. Again, only the team members may contribute to the solution through their knowledge and work, and by using inanimate resources (e.g. research articles, websites, textbooks, journals, and publications). You will also need to cite these inanimate resources in your reference or works cited section. Additionally, COMAP will never require that you purchase additional materials or information to be successful in the HiMCM. The materials and information provided by COMAP, along with your own team's knowledge, skills, and perhaps a bit of research using allowed references, is all that you need for success.

COMAP uses Twitter and Weibo to provide contest information to participants. Follow us @COMAPMath on Twitter or COMAPCHINAOFFICIAL on Weibo for contest guidance and up to date contest information.

Judging

All contest submissions are electronic. This allows a high quality and diverse judging pool from academia and industry to simultaneously judge papers. Soon after the contest ended, we conducted our first round of contest judging with a diverse set of judges from the United States. Each paper is read and scored by two preliminary judges. We thank these judges for their careful review of our HiMCM submissions.

All judging is blind with respect to any identifying information about the participants or their schools. Each year during the contest, COMAP receives a few clarifying questions about the problems. In most cases, our response is the same: these are open-ended questions and based on your assumptions and approach your team will do its best to answer the questions posed. Your responses are not compared to an official answer key since based on the path(s) your team took will lead to potentially vastly different and equally correct solutions. Each submission is scored on its own merits as we look for completeness, creativity, and the use of good math modeling techniques. Preliminary judges rank papers as Finalist, Meritorious, Honorable Mention, and Successful Participant. Judges sent all papers ranked as "Finalist" to Final Judging. This year, 67 papers from the two problems went to Final Judging for a panel of twelve judges to consider. As these 67 papers were the best submissions from the preliminary round, at final judging the judges chose the "best of the best" as Outstanding papers. Nine papers earned the Outstanding award. The final judges commend the preliminary judges for their efforts in selecting the high-quality Finalist papers. We feel that the structure of preliminary and final judging provides a good process for identifying our top papers.

The Future



For 25 years, the HiMCM has sought to provide all high school students the opportunity to compete and achieve success in applying mathematics. Our efforts remain focused on meeting this important goal. Mathematical modeling continues to grow within the high school curricula across the globe, and we recognize that middle school students are now modeling too. In 2023, COMAP held its third international MidMCM, a middle school/level contest option. The MidMCM occurred concurrently with HiMCM. The MidMCM allows middle school/level students under the age of 14 ¹/₂ years old the opportunity to demonstrate their mathematics and modeling



abilities. Please visit <u>www.MidMCM.com</u> for more details and the results of this new contest.



The MidMCM and the HiMCM provide a vehicle for using mathematics to build models that allow students to represent, and to understand, real world behavior in a quantitative way. Both contests enable student teams to look for patterns and think logically about mathematics and its role as a language in our daily lives. Students gain confidence by tackling ill-defined problems and working as part of a team to generate a solution. We are excited that in our contests, applying mathematics is a team sport.

Advisors and students often ask what level of mathematics is required, and what special programming or coding skills are needed for the contest. All our HiMCM problems are accessible using high school level mathematics alone, and no programming or coding skills are required or necessary. Our new MidMCM problems require only middle school/level mathematics. As in all our contests, each of our problems is accessible on multiple levels. Students should apply the mathematics they understand and are able to explain in their solution analysis. COMAP recognizes that there is no single standard or body of knowledge of High School level mathematics for an international math modeling competition. The judges see a wide range of abilities and skills. Teams should note that using so-called lower-level mathematics effectively can be just as successful as employing higher-level mathematics in the competition. Rather than attempting to impress the judges with overly complex techniques and models, teams should concentrate on employing mathematical techniques they are familiar with and using sound modeling techniques to develop their solutions.

Advisors need only be motivators and facilitators to encourage students to be creative and imaginative. COMAP

invites all middle and high school mathematics faculty to get involved, encourage their students to be problem solvers, make mathematics relevant, and open the doors to future success. We aim to partner with teachers to continually improve the contest to make it accessible and impactful to all students. Any school can participate, and each school can enter as many teams as that school desires. MidMCM and HiMCM have no restrictions on the number of total schools or the numbers of total teams. Advisors should encourage student teams to explore the COMAP website for resources and read judges' commentary on past student solutions. More than just learning skills and operations, mathematics is both an art and a science. Through mathematical modeling, students learn to think critically, communicate effectively, and become confident, competent problem solvers.

2024 Contest Dates

Mark your calendars for the next HiMCM, and the fourth annual MidMCM, to be held November 6 - 19, 2024. Registration for the 2024 MidMCM and HiMCM

will open in September. Student teams may work at any time during the contest window. At the team members' convenience, teams download and choose their problem, complete their modeling solution, and electronically submit their solution document by the deadline on November 19th. Again in 2024, one team for each problem will receive the NCTM award. Teams can learn more about COMAP's contests and registration at www.comap.org

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MATHmodels.org

Powered by COMAP content, MATHmodels.org is a wonderful resource for students and teachers to make math modeling a year-round activity. Teachers and students may use the materials found on this site to enrich their classes and help prepare students for mathematical modeling competitions. We encourage you to visit

www.mathmodels.org.



After final judging, HiMCM papers receive a designation in one of the categories above. Depending upon the quality of the papers, the top 20–25% of submitted papers receive a designation of Meritorious or above, with approximately the top 1% designated as Outstanding.





The International Mathematical Modeling Challenge, IM2C

The IM²C is held each spring and continues to grow. This unique contest is similar to an all-star game where each country/region administers and judges the contest and then sends its top two teams to the international final judging. The purpose of the IM²C is to promote the teaching of mathematical modeling and applications at school level (high school and below) for all students around the world. It is based on the firm belief that students and teachers need to experience the power of mathematics to help better understand, analyze, and solve real world problems outside of mathematics itself - and to do so in realistic contexts. An international Expert Panel of final judges determines winners and selects teams to present their solutions at an international award ceremony. To learn more visit www.immchallenge.org for rules and country/region contacts.

COMAP invites selected teams from the United States, to include teams earning Meritorious or above in the HiMCM contest, to compete in the IM²C U.S. Regional Round. Registration is free. From these participants, U.S. IM²C judges select the two top teams to represent the U.S. in the IM²C international round. U.S. Rules at: https://immchallenge.org/Pages/Rules/ USA/USA-Rules.html

Problem Discussions and Judge's Commentary

The following paragraph describes what our preliminary and final judges look for in our HiMCM papers:

Regardless of the problem chosen, competitive papers include a comprehensive summary, address all requirements through developing and applying a mathematical model(s). Better papers do all the above in an articulate, well-supported, well-organized, and well-presented manner. The best papers combine complete mathematical and logical analysis and explain their work in an organized presentation beyond simply addressing the requirements. The best papers are also easy to read, creative, flow logically, and include sections that address assumptions with justifications, the modeling process(es), results of modeling and analysis, strengths and weaknesses, sensitivity, conclusions, and references.

Our judges have asked that we continue to stress that all our HiMCM problems are accessible by students at any level using high school mathematics. Some teams attempt to use advanced concepts and tools found on the Internet that they do not explain clearly or use appropriately. Judges recognize this, and these papers do not do well. We are not looking for papers that use the most advanced mathematics. We have found that the best papers develop a mathematical model incorporating high school level concepts and tools that teams understand, can fully explain, use appropriately, and for which they can contextualize and analyze the results. The most important aspects of solutions are the model development and the clear use and analysis of the model toward addressing the requirements of the problem.

The specific problem discussions below provide comments on how teams addressed the requirements of each problem. Following this section, we provide judges' comments about the solutions and presentations by breaking down the various parts of a submission and providing exemplars. To view the complete problem statements, visit

www.mathmodels.org or www.himcmcontest.org.

Problem A: Dandelions: Friend? Foe? Both? Neither?



This year's Problem A challenged teams to explore the multifaceted roles of dandelions, a plant native to Eurasia that has since spread worldwide. Commonly seen as a weed, dandelions are recognized by their distinctive yellow flowers and fluffy seed heads that disperse seeds by wind. Yet, beyond being a nuisance to some in their gardens, dandelions possess significant culinary and medicinal properties, emphasizing the plant's conflicted nature as both invasive and beneficial.

Teams were tasked to develop two main mathematical models. The first model was to predict the spread of dandelions over time across a onehectare plot under various climatic conditions including temperate, arid, and tropical environments. This encouraged teams to research seed dispersal mechanisms influenced by different weather patterns and geographical features.

The second requirement was to create a model for calculating an 'impact factor'





for invasive species. Initially applied to dandelions, this model incorporated variables such as species characteristics, environmental impacts, and interactions with other flora and fauna. Teams were then required to extend this model to assess the impact of two additional invasive plant species of their choice, thereby assessing the model's versatility and applicability in different ecological contexts. This part of the task emphasized the adaptability of their model in various regions and conditions, highlighting the critical need for robust models that can manage and predict the consequences of invasive species in diverse ecosystems.

In addition to their mathematical work, students were encouraged to consider the broader ecological and social implications of invasive species. The problem underscored the importance of understanding the delicate balance between a species' benefits and its potential threats to ecosystems.

The submissions displayed a range of innovative approaches and solutions, reflecting deep engagement with the complex dynamics of ecological modeling. This year, teams were particularly noted for their ability to integrate detailed research with practical modeling techniques. This resulted in the development of quantitative tools that, given a wide variety of different inputs, provided multiple methods to compare plant species and rate their invasiveness. The judges enjoyed reading these thoughtful and creative solution papers.

Problem B: Charging Ahead with E-buses



In this year's Problem B, the focus shifts to the accelerating adoption of electronic

buses (e-buses) in urban environments globally, a crucial step toward sustainable urban transportation. As cities grapple with air pollution and climate change, the transition from diesel to electric buses represents both an environmental imperative and a financial challenge. This problem draws attention to the significant uptick in e-bus deployment, highlighted by China's policies favoring electric vehicles, setting a benchmark for countries, regions, and cities worldwide. Participants were tasked with multiple objectives centered around the integration of e-buses into city fleets.

The first challenge was to develop a model that could quantify the ecological benefits of transitioning to an all-electric bus fleet. This model needed to consider a specific metropolitan area with over 500,000 inhabitants, which hasn't yet converted to a fully electric fleet, highlighting the model's practical implications.

Next, teams were required to address the economic aspects of such a transition through a second model that incorporates potential financial support, such as grants and subsidies, which could cover up to 50% of the initial costs. This model, which was to be applied to the same metropolitan area they identified in the first prompt, needed to evaluate the long-term financial sustainability of adopting e-buses, considering high upfront costs and operational savings.

The third prompt asked teams to propose a 10-year roadmap for their chosen metropolitan area to achieve a fully electric fleet by 2033. This involved scaling their ecological and financial models to forecast the gradual replacement of diesel buses, taking into account various logistical and strategic challenges.

Lastly, the participants were to compile their findings and recommendations into a concise, one-page letter addressed to transportation officials of the metropolitan area. This letter was intended to summarize their recommendations and serve as a direct communication to help guide policy and decision-making processes.

As in previous years, the judges were impressed with the depth and quality of work teams demonstrated in developing and presenting their models. Teams considered a wide variety of possible ecological aspects to measure the benefits, or consequences, of transitioning to an electronic fleet of buses, including the impact of various greenhouse gases and noise pollution along the bus routes. Many teams went a step further by evaluating the ecological impact of the manufacturing process of new buses. For the financial analysis, teams generally employed curve-fitting and other time-series analysis techniques to forecast costs. These approaches provided a nice segue into the third part, where teams proposed replacement plans. The stronger teams considered multiple approaches from which they could identify an optimal plan. Many teams recognized the geospatial nature of the problem, illustrating specific routes to be replaced and identifying locations for charging stations and other necessary infrastructure.

Judges' Discussion

While the problem discussions above provide comments on the solutions to this year's problems, in the following paragraphs we examine the sections of a submission and provide comments about the solutions and the presentation of the solutions from our judges' point of view. At the end of the article, we have included excerpts from papers that earned Finalist and Outstanding designations as exemplars for both problems. Mathmodels.org members can view the full, unabridged versions of these papers online.

Overall

Student participants must ensure their papers follow the contest rules posted on the contest website. Every year, judges encounter papers that exceed the page limit, use smaller fonts, or widen margins to include more material.



While these papers are not necessarily disqualified, they are unlikely to score well. The best papers are coherent, organized, clear, and well-written, leaving a great impression on the judges. The logic and mathematics in these papers are easy to follow, presenting an understandable sequence and a clear story. These papers thoroughly explain the background and justification of the chosen model(s) and apply them properly. Teams should present their entire submission in 25 pages or less, using at least a 12-point font. These 25 pages should include an introduction/ executive summary, a solution that addresses all requirements, a resource list, and any appendices. While some background research on the problem topic can be included, it should be brief. Recognition is based not on the number of pages but on the ability to complete all problem requirements and communicate the solution concisely and clearly. Students should use spelling and grammar checkers before submitting their papers. Papers from non-English-speaking countries/regions should ensure all symbols in tables and graphs are in English. Student and school names should not appear on solution papers.

Papers considered for Finalist and Outstanding start with a clear summary that describes the problem. These papers then preview their paper with an organized Table of Contents. They present assumptions with justifications, explain the development of their model and its solutions, apply their model, and support the results mathematically. The top papers communicate all the listed aspects, perform a sensitivity analysis, address model strengths and limitations, and finally, close by stating overall conclusions.

Lastly, please submit your solution one time only and clearly indicate whether you are submitting Problem A or Problem B. Making multiple submissions or failing to specify your chosen problem will slow down the judging process.

Executive Summary

The Executive Summary, often the first page or cover sheet, provides a first impression of your paper. It offers the judge (or any reader) not only a synopsis of the paper and the modeling and analysis process but also the solution(s) to the problem. Judges often see welldetailed descriptions of the problem and process but are particularly impressed by well-written and complete summaries that include the actual results and recommendations. Unfortunately, many submissions each year fail to include results in their Executive Summary. Alternatively, some submissions include results but fail to contextualize them. For example, if your model outputs a score of 17, it is meaningless without context, such as stating that the model's score is twice as good as a baseline (or some other contextually relevant comparison).

Teams should write the Executive Summary after completing their solution to summarize the entire contents of the paper. Example 1 from Team #14632's submission for Problem A demonstrates a fine example of a summary, including results rather than just a discussion of their approaches. An ideal summary can stand alone, providing the reader with a synopsis of the problem, the methods used, and the solutions. Similarly, Example 2 from Team #13681 was well-written and stood out to the judges, presenting results, and providing context to highlight the benefits of the team's modeling approach for Problem B.

Assumptions with Justifications

Effective mathematical models incorporate a few essential assumptions to simplify the modeling process. These are often referred to as simplifying assumptions. A common mistake seen by judges is the inclusion of unnecessary or irrelevant assumptions. When developing a model, students should ask themselves, "do we need this assumption to apply our model"? Identifying and clearly stating good and relevant assumptions can be challenging. Long lists of assumptions that do not directly contribute to the model's development or solution can detract from the paper's quality. Moreover, assumptions that overly simplify the problem may prevent a comprehensive solution. Each assumption should be accompanied by a brief justification demonstrating that it is both reasonable and necessary. These assumptions often directly relate to your model sensitivity analysis or impact your model strengths and weaknesses. For example, if you assumed a certain value or no impact by a certain exterior factor, how would changing this assumption impact the validity of your model conclusions? Teams that do well recognize the extent to which their assumptions limit and shape their model, and they provide an upfront and transparent analysis of how changing some assumptions might impact their model and results. See Example 3 for a concise and well written set of assumptions with justifications of Problem B from Team #14048. The team cited external resources to clearly define the scope of their model in two ways: (1) specifying which greenhouse gas emissions would be used in evaluating their model, and (2) specifying which processes in the manufacturing process could be ignored due to negligible emissions.

Definition and Use of Variables

Most mathematical models include several variables that teams must define for the reader. This list of variables should include the variable symbol, a brief description of the variable, and the units of the variable. Judges often see lists of 10, 20, or even 30 or more unique variables in HiMCM submissions. There is no minimum or maximum number of variables the judges are looking for, but rather did the students choose an adequate set of variables. Any complexity gained by adding an excessive number of variables is often offset by their practicality and usefulness. Using best practices, teams should focus on a manageable set of variables when modeling. Judges often see many



variables that are defined but never used in the models. Additionally, as you use variables in your model, remind the reader of the variable definitions and units. This practice assists the reader in following the logic of your process. A nice list of variables for Problem A is shown by team #14632 in **Example** 4. Their list both defines the variables and includes units of measurement where applicable. In Example 5, Team #13694 for Problem B similarly provided concise lists of variables used before each section and would add a list of additional variables used at the start of the relevant sections they appeared.

Mathematical Model

The development of the mathematical model is the most important part of your submission. There is always more than one appropriate solution method to our HiMCM problems and so teams should address the problem with the mathematics they know and understand. Papers should explain the development of the mathematical model(s) and/or algorithm(s) and define all variables in a logical manner. Teams should take the reader on a journey describing why they selected a particular model or decided to use one or more models. Better teams will explain why they choose their model and how they plan to use or modify it to fit this problem. Teams that merely present a model without explaining or showing the development of that model do not generally do well. Although during your HiMCM work you may develop several models, presenting multiple models without identifying the most appropriate model to answer the questions is detrimental to your paper's success. Judges are more impressed with a well thought out (and perhaps simple) model than with an overly complex model that a team struggles to apply. To impress the judges, focus on applying sound principles to your model that you understand. Judges do value creativity and thinking "outside of the box" in your modeling process but be sure to balance creativity with your level of expertise and modeling experience. Judges continue to reward creativity. A submission that searched the internet for an existing model to apply may not do as well as a team who modified an existing model to address the situation, or took the results from one model as inputs to another model. So, be creative and have fun.

Perhaps the most important step of the modeling process is the last one: explicitly present your final model in its full form. Do not make the judges have to look for your final model. Judges continue to see papers with an initial model mentioned in one section and then different models used in subsequent sections without the team connecting the models logically. Be consistent and logical with your modeling process and guide the reader through your solution. Clearly identify your model(s) and then use your final model as you address the problem requirements and determine your results. Does your model use the variables and the assumptions you made earlier in your submission? Papers that do not flow well tend to not be judged as the better and best papers.

There are many ways to model and analyze the HiMCM problems. This year we saw a variety of appropriate, as well as creative, models to address each problem. The use of tables, graphs, and images is often helpful to show your modeling process. Judges appreciate a good mix of visual aids and quality writing. Papers that are pages and pages of text, or on the contrary are pages of graphs and charts with minimal text, do not do as well as others that have a good mix of both. The better papers will show a graph and chart and then immediately discuss the impact of this visual. We include several examples of the processes involved in model development for this year's problems. For the first part of Problem A, the Dandelion Problem, Team #14076 in Example 6 creates an accessible transition diagram that provides a graphical basis for the team's approach to modeling the potential spread of a single dandelion. The subsequent discussion steps the reader through the mathematics used to return one possible outcome from their model.

Another approach that teams are encouraged to explore is the use of provided data to help develop all parts of their model. In Problem B, Team #14457 creatively used some statistical tools with all the available data to identify three types of bus route behavior, which then formed the basis for their model; see **Example 7**.

While not necessary, teams may use technical computing to enhance their models and results. One example that stood out to judges is highlighted in Example 8, where Team #13795 used the graphical simulation software Simulink to run simulations of their model for Problem B. Teams are encouraged to be creative with the computational tools they use. Those who can master a tool, use it effectively, and provide justifiable reasons for its use have the potential to stand out. However, we want to reiterate that using computational software is not necessary for teams to do well in the HiMCM.

The second part of Problem A was to create a model for calculating an 'impact factor' for potentially invasive plant species. Like all parts of the contest, judges do not compare student's specific answers to a solution key. We are not looking for a particular answer. Judges are more interested in the assumptions and models chosen by the teams. We read a variety of responses, with some teams identifying dandelions as being more or less invasive and determining impact factors for numerous different plants. If the assumptions and other factors are properly justified, most solutions can be acceptable. In Example 9, Team #14211 effectively prepares the reader to understand their model by providing a thoughtful and thorough discussion of risk indicators for invasive species and their scoring criteria.

For the final part of Problem B, teams were asked to devise a 10-year roadmap to electrify bus routes in their chosen



cities. Throughout all submissions, teams effectively used their models from earlier parts of the problem to analyze what each year of electrification would entail. Given the real-world and geographic nature of the problem, many teams went on to explicitly show what their plans would look like for each city. Team #14087 produced annotated maps showing which bus routes in their three chosen cities would be electrified, along with when in the 10 years each would be replaced; this is provided in **Example 10**.

Sensitivity Analysis

Sensitivity analysis is an important part of the modeling process. Its primary purpose is to test some of the assumptions made, including parameter values. If your model yields certain results based on these assumptions, what happens if you modify an assumption? Example 11 from Team #13753 provides an extensive look into changes in parameters such as temperature, precipitation, wind velocity, and wind directions as part of their solution to Problem A. This type of analysis was well-received by the judges. However, it should be noted that space limitations may not allow for such a substantial response. In this instance, the judges appreciated that the sensitivity analysis was accompanied by a brief discussion on the impact on the solution. This discussion can justify the use of additional space, as it naturally addresses the strengths and weaknesses of the model.

While many teams save their sensitivity analysis for the end, some incorporate this analysis throughout as pieces of their model come together. **Example 12** illustrates this approach as part of Team #14323's model development for Problem B.

Strengths and Limitations

Given the limited time teams have to develop their models, it is expected that each model will exhibit both strengths and limitations. Teams should critically evaluate their proposed models and solutions, addressing strengths, limitations, and potential extensions or improvements. Is your solution reasonable? Under what conditions will it perform best, and where will it fall short? If more time or resources were available, what additional information would the team need to improve their model? This section is also an opportunity for teams to apply common sense and assess the reasonableness of their proposed solutions. **Example 13** showcases a detailed discussion of the strengths and weaknesses of Team #14555's models for Problem B.

Conclusion

Many students confuse the conclusion with the Executive Summary. The conclusion wraps up the report, summing up the argument or providing the team's opinion or judgment. In contrast, the Executive Summary is a stand-alone section that allows a reader to quickly understand the entire problem, process, and solution. Team #14140's conclusion, as shown in Example 14, effectively reminds readers of their process and reports results throughout their submission for problem A. As a result, their concluding remarks were especially readable and served as an impactful reminder of their overall presentation.

The Infographic, Blog, or Non-technical Article

The purpose of this section is to demonstrate to the judges that you can convey your work and solution in common terms, regardless of the complexity of your mathematics or analysis. Often, the recipients and consumers of mathematical modeling and research are not as technically astute as the mathematicians and scientists doing the work. Therefore, teams must translate their efforts into a shorter, descriptive format, such as a news article or graphic. This increases the likelihood that your audience will understand and seriously consider using your solution and recommendations. This section should tell the same story as your technical write-up but in a more accessible way, using straightforward writing and graphics.

For instance, **Example 15**, a non-technical article for Problem B, clearly presents Team #14789's findings in an easy-to-understand manner suitable for stake-holders.

Although Problem A did not specifically ask for teams to write a letter to a client, there were still many examples of informative graphics found throughout the submitted papers. A nice example is shown in **Example 16** from Team #13719, who shared a map with pins identifying the locations they examined while modeling dandelion growth.

Citations and References

Citations and references are crucial in these papers and any other papers that use outside sources. Teams using existing models should cite their sources at the point they present the model and include a reference citation at the end of the paper. This applies to all graphs and tables taken from the literature as well. Use in-line documentation with footnotes or endnotes to give proper credit to outside sources. All data, figures, graphs, and tables from outside sources require documentation at the point where they appear in the paper. Lack of documentation will result in a lower designation.

We have noticed an increase in the use of tertiary sources such as Wikipedia. While useful, information from Wikipedia might not always be accurate. Teams should recognize this and seek out primary sources such as research journal articles, government websites, and other reputable sources.

AI Usage

An Artificial Intelligence (AI) Usage Report is not a required element of the presentation, but if Large Language Models (LLMs) are utilized while building solutions, they need to be referenced. As noted in the contest guidance regarding LLMs, "It is important to note that LLMs and generative AI have limitations and are unable to replace human creativity and critical thinking." The contest recognizes that



AI and large language models are reasonable tools to utilize while developing a modeling solution, but it is essential for teams to reference their use. The AI usage reports from Team #14209 (Problem A) and Team #13921 (Problem B), found in **Examples 17 and 18** respectively, more than satisfied this requirement. Both teams shared their reports in readable formats, clearly identifying how LLMs were incorporated into their solution approaches.

Final Thoughts

On behalf of COMAP and the contest judges, we would like to again thank all advisors students for their participation in the HiMCM. Each year the quality and level of mathematics demonstrated by our high school student teams amazes and impresses our judges. We genuinely enjoy reading all solution papers. Successful teams use a wide variety and level of mathematics. While teams using post-high school/undergraduate level mathematics are in a league of their own if they understand and can explain their work, teams using basic high school mathematics and much simpler approaches are often among our Outstanding designees. We encourage students of all levels to compete in future HiMCM competitions as well as our MCM/ICM contests targeted to undergraduates. To be successful, read the comments and guidance provided in this article, see the TIPS article on the COMAP website, and visit mathmodels.org to review previous problems.

Follow us @COMAPMath on Twitter or COMAPCHINAOFFICIAL on Weibo, LinkedIn, and/or Facebook for information about all COMAP contests.

ry (Problem A, Team 14632, 10

- 1. Summary (Problem A, Team 14632, Mass Academy of Math & Science, MA, USA)
- **2. Summary** (Problem B, Team 13681, Nanjing Foreign Language School, China)
- **3. Assumptions** (Problem B, Team 14048, The High School Affiliated to Renmin University of China, China)
- 4. Variables (Problem A, Team 14632, Mass Academy of Math & Science, MA, USA)
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Team Control Number 13681 Problem Chosen B 2023 HiMCM/MidMCM Summary Sheet

Driving Towards a Greener Future

Given the escalating concerns surrounding environmental issues and the energy crisis, numerous cities worldwide have embraced sustainable urban transportation solutions, such as electric buses. This study evaluates the ecological and financial implications of transitioning from diesel to electric buses and proposes a detailed transition roadmap.

To assess the environmental impact of adopting an all-electric bus fleet, we first identify three aspects that serve as indicators of the environmental impact: energy consumption, air pollution, and noise pollution. We develop the 'EAN Model' to integrate these aspects, using metrics like equivalent oil consumption, air pollutant emissions, and noise levels. Our analysis, applied to New York City, reveals that electric buses are significantly more energy-efficient (8.93 times less energy consumption), emit fewer nitrogen oxides and particulate matter, and are quieter (noise levels below 60 dB compared to over 90 dB for diesel buses). These findings strongly advocate for the ecological benefits of a full e-bus fleet transition.

In order to analyze the financial implication of changing to electrical buses, we identify four aspects that constitute financial implication: maintenance cost, acquisition cost, infrastructure cost, and external funding. We develop the 'MAIE Model' to aggregate these financial aspects. Our case study on New York City suggests an initial investment of approximately \$2.17 billion but forecasts annual maintenance cost savings of around \$14.35 million post-transition. Despite the significant upfront costs, the long-term financial and ecological advantages underscore the necessity of transitioning to a full e-bus fleet.

To effectively assist cities in transitioning to electric buses, we have developed a comprehensive 10-year roadmap, segmented into a 2-year Exploratory Launch Phase, a 5-year Strategic Expansion Phase, and a 3-year Full Electrification Phase. To optimize the percentage during transition, we approach it as an optimization problem, balancing the objectives of maximizing ecological benefits and minimizing financial expenses. By transforming these considerations into a single-objective optimization problem, we employ a linear programming model to derive the optimal strategy that maximizes benefits while adhering to budget limitations. We then applied our model to New York, Hong Kong, and Bogota to show the different transition roadmaps of cities with distinct characteristics.

Finally, a sensitivity analysis on the linear programming model was conducted, alongside a detailed evaluation of the investigation's strengths and weaknesses. We also wrote a letter to transportation officials in New York to outline the benefits of adopting our 10-year electric bus transition roadmap.

Keywords: Electrical buses; Diesel buses; Linear Programming; Transition Roadmap; Ecology; Finance

Example 2: Summary, Problem B Team #13681, Nanjing Foreign Language School, China



Team # 14048	Page 4 of 25
2.2 Assumptions	
We made the following assumptions in our mode	el:
 To get more detailed analysis results, we split the according to the International Council on Clear 	he bus GHG emissions into 4 parts n Transportation (ICCT) report ^[4] :
 Diesel bus GHG emissions come from fumaintenance, and vehicle manufacture. Electric bus GHG emissions come from elevehicle manufacture, and battery manufacture. 	ectricity production, maintenance, cture.
GHG emission of vehicle/battery manufacture tery is purchased.	will be calculated when bus/bat-
The money earned from the operation is the s and the replacement ratio of diesel buses to e-b	ame for diesel buses and e-buses, buses is 1:1.
3. Prior to the transitions, the age distribution o uniform. More precisely, if the lifespan of a bu at each age from 0 to 11 was consistent.	f the existing diesel bus fleet was s is 12 years, the quantity of buses
According to [5], GHG emission is relatively l portation and installation of charging stations chargers, so we neglect them to simplify the with the operation of charging stations overlap considered, so we do not take them into account	ow during the production, trans- and the recycling process of e-bus model. The emissions associated with other sectors we've already ht.

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2.3 Brief Overview

TEAM 14632

First, we model the spread of the seeds from an initial puffball across a one-hectare plot of land. Afterward, we minimize the scope of the problem, focusing on each individual seed to model the independent growth of a seed. Once each seed matures into a puffball, the process repeats.

To model the life cycles of a seed, we developed a *Seed Agent Model*, which considers environmental characteristics, such as wind, temperature, sunlight, and nutrients, to iterate each seed through a germination and plant growth life cycle. Finally, when new puffballs are fully developed, the Seed Agent Model is again applied to model the spread and growth of every new seed. Throughout our process, we incorporate environmental unpredictability by developing several stochastic processes.

Below is a table of variables used throughout our agent-based model.

Variable	\mathbf{Symbol}	Description
Time Step	t	Each time step is a day where $t = 0$ is January 1 of a year.
Number of Moves	N	Number of moves a seed takes in its movement process.
Coordinate Shift	$\Delta x, \Delta y$	Change in x or y coordinate of a seed in every move.
Death Date	D	Time of death of a seed, sampled from an Exponential distribution.
Plant Growth	G	The stage of growth of a dandelion plant between 0 and 1.
Temperature Index	T_{I}	A standardized metric to describe optimal temperatures for dandelions.
Light Index	L_I	A standardized metric to describe optimal light levels for dandelions.
Nutrition Index	N_I	A standardized metric to describe optimal soil conditions for dandelions.



Team #13694, Page 13 of 25 **5** Financial Modelling (Requirement 2) 5.1 Variables We reuse relevant variables from our ecological model (requirement 1), and define additional ones below. Definition Variable Variable Definition *S%* Td % of costs subsidized by the distance of trolley infrastructure present government F_x financial cost of process xМ maintenance cost per km of trolley infrastructure Fc_f fuel cost per unit of fuel f P_{bus} buy price of bus Yc_f yearly delivery charge of fuel f(if V_{bus} scrap value of bus applicable) E_p cost of purchasing part pB_{battery} battery capacity of battery

> Example 5: Variables, Problem B Team 13694, St. George's School, Canada



Team Control Number: 14076

2.2 Estimating the population growth using Leslie matrix

The dandelion life cycle from seedling to mature seed head spans 3-4 months. We can divide this 12-month period into 4 states: S_0, S_1, S_2, B . S_0 represents a new seedling. S_1 and S_2 represent seedlings grown for 1 and 2 months, respectively. *B* denotes a blossomed flower that will transition into the puffball stage to propagate seeds in the next month, completing the cycle. The transition between these four states is illustrated in Figure 1.



Figure 1 Transition diagram of the four states of a dandelion The information in the transition diagram can be integrated into a transition matrix **T**. This matrix is also known as the Leslie matrix ^[8], and it is widely used to represent the transition between different age groups of a species. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & N \end{pmatrix}$

	/ 0	0	0	N
$\mathbf{T} = (\mathbf{t})$	1-p	0	0	0)
$\mathbf{I} = (t_{ij}) =$	p	1-p	0	0
	0	p	1	0/

The first row represents the "fecundity" of each state. Since only a blossom can produce a new seedling, the only non-zero element is $t_{14} = N$, indicating a blossom can produce N alive seedlings in the next month, after taking into account the total number of seeds in a puffball and the survival rate of them.

The first column of **T** shows that a proportion of (1 - p) of the seedlings in state S_0 grow into state S_1 , while a proportion p of them are advanced into S_2 in the next month. Similarly, the second column indicates the same proportion of dandelions in state S_1 , which directly develop into blossom B and skip the state S_2 . The only non-zero element in the third row $t_{43} = 1$ means that a dandelion in state S_2 is certain to grow into a blossom. This certainty guarantees that a seedling can accelerate at most once during its life cycle, either from S_0 to S_1 or from S_1 to S_2 , so that the minimum life cycle is 3 months while the normal life cycle is 4 months. The overall probability that a seedling has life cycle of 4 months is $(1 - p)^2$, and the probability of an accelerated life cycle is

$$P_{short} = 1 - (1 - p)^2$$

Therefore, the value of p in matrix **T** can be calculated as

$$p = 1 - \sqrt{1 - P_{short}}$$

In our simulation, $p = 1 - \sqrt{1 - 0.25} \approx 0.134$.

Example 6: Model Development, Problem A Team #14076, Shanghai Pinghe School, China



Team Control Number: 14076

The state matrix \mathbf{s}_n is a column matrix showing the number of dandelions in each state n months after the spread of the initial seed head.

$$\mathbf{s}_n = \begin{pmatrix} q_{0,n} \\ q_{1,n} \\ q_{2,n} \\ q_{b,n} \end{pmatrix}$$

where $q_{0,n}, q_{1,n}, q_{2,n}, q_{b,n}$ represents the amount of S_0, S_1, S_2, B after n months respectively.

The state matrices of two consecutive months can be related by the matrix multiplication

$$\mathbf{s}_{n+1} = \mathbf{T}\mathbf{s}_n$$

$$N \cdot q_{b,n}$$

$$(1-p) \cdot q_{1,n}$$

$$\left(p \cdot q_{0,n} + (1-p)q_{1,n} \right)$$

$$\left(p \cdot q_{1,n} + (1-p)q_{2,n} \right)$$

Assume the parent seed head propagates its seeds at the beginning of a 12-month period. The initial state is

$$\mathbf{s}_0 = \mathbf{T} \begin{pmatrix} 0\\0\\0\\1 \end{pmatrix} = \begin{pmatrix} N\\0\\0\\0 \end{pmatrix}$$

meaning that a total of N seedlings are present initially.

Then the state matrix after n months can be derived by repeatedly pre-multiplying the Leslie matrix \mathbf{T}

 $\mathbf{s}_n = \mathbf{T}^n \mathbf{s}_0$

Substituting the values $p = 1 - \sqrt{0.75}$, N = 5, we obtain

$$\mathbf{s}_{1} = \mathbf{T}^{1}\mathbf{s}_{0} = \begin{pmatrix} 0\\ 4.33\\ 0.67\\ 0 \end{pmatrix}, \qquad \mathbf{s}_{2} = \mathbf{T}^{2}\mathbf{s}_{0} = \begin{pmatrix} 0\\ 0\\ 3.75\\ 1.25 \end{pmatrix}, \qquad \mathbf{s}_{3} = \mathbf{T}^{3}\mathbf{s}_{0} = \begin{pmatrix} 6.25\\ 0\\ 0\\ 3.75 \end{pmatrix}$$
$$\mathbf{s}_{6} = \mathbf{T}^{4}\mathbf{s}_{0} = \begin{pmatrix} 7.81\\ 0\\ 14.06\\ 9.38 \end{pmatrix}, \qquad \mathbf{s}_{9} = \mathbf{T}^{6}\mathbf{s}_{0} = \begin{pmatrix} 9.77\\ 60.89\\ 44.58\\ 17.58 \end{pmatrix}, \qquad \mathbf{s}_{12} = \mathbf{T}^{12}\mathbf{s}_{0} = \begin{pmatrix} 275.88\\ 228.35\\ 101.24\\ 29.3 \end{pmatrix}$$

The final total number of dandelions of four states is

 $275.88 + 228.35 + 101.24 + 29.3 \approx 635$

which indicates a single seed head can produce a total of 635 seeds after 12 months. Our simulation result is

comparable with the reported statistic in ^[1, 4, 5], taking into account a survival rate of approximately 2-4%.

Example 6: Model Development, Problem A, Continued Team #14076, Shanghai Pinghe School, China







Team # 14457 Page 9 of 23 2). Combine the clusters with the smallest distance into 1 cluster (Combined CF) and calculate the distance between the new cluster and the remaining clusters. 3). The closest clusters are further merged, and if the threshold is exceeded, they are split and balanced. 4) Repeat Step 2 and Step 3 until all clusters are fully merged to form a nested tree structure. Since the electric buses only run in the urban area, the kinematic segments can be classified into three categories: crowded, average and smooth. In python environment, the kinematic segments are clustered according to the three categories, and the results of clustering using Birch and K-means are shown in Figure 1 for Route 362, for example. **Birch Clustering** 0 -1 -2 -3 -0.5 -1.0 0.0 0.5 1.0 1.5 2.0 2.5 3.0 Figure 1 Classification results 3.1.3 Carbon emission reduction accounting model for purely electric public transportation vehicles Pure electric buses are powered by electricity and do not produce direct carbon emissions while running, and were once thought to be "zero-emission". However, a country's power structure may be dominated by thermal power generation, which generates large amounts of greenhouse gas emissions. If we consider the carbon emissions from electricity production from a full life cycle perspective, the operation of pure electric buses is far from carbon zero. Based on this understanding, we developed a full life cycle carbon emission reduction model for pure electric buses based on the existing power structure in China, and used it to analyze the different types of buses mentioned above. 1). Baseline emissions model The baseline scenario refers to the carbon emissions that would have been generated if the same level and characteristics of products or services had been provided in the absence of the emission reduction project, and is the precondition for the emission reductions generated by the voluntary emission reduction project, which is essentially a reference system. For the electric bus

> Example 7: Model Development, Problem B Team #14457, Affiliated Middle School to Jilin University, China





Figure 9. Main Part of Differential System in SIMULINK

3.2.2 Government Subsidies (External funding)

Because fiscal subsidies are the most important external funding for bus company, we do not differentiate between external funding and fiscal subsidies in the paper.

Given the requirement that potential external funding mainly including government subsidies in the model, cover no more than half of the transition costs, we reach the following equation.

 $\frac{1}{2}C_{trans,max} \ge g \ge 0$

Note. $C_{trans,max}$ reflects the maximum cost of transition at any point in time during the transitioning process.

On this basis, because we assume that the private entities involved would not lose funding by the end of the transition, there is another extra requirement for government subsidies, as indicated below. In this requirement, government subsidies g_{total} include all exclusive fundings such as government subsidies without transition, extra government subsidies for transition g_{trans} .

$$R_{trans,cp}(T) + g_{total} \ge 0$$

We then rearrange it into:

 $\frac{1}{2}C_{trans,max} \ge g_{total} \ge -R_{trans,cp}(T)$

Moreover, to guarantee that there are financial gains as a result of this transition, the following requirement should also ideally be fulfilled. In this requirement, government subsidies g_{trans} only include extra government subsidies for transition.

$$R_{trans,cp}(T) + g_{trans} \ge R_{org,cp}(T)$$

We then rearrange it into:

$$\frac{1}{2}C_{trans,max} \ge g_{trans} \ge R_{org,cp}(T) - R_{trans,cp}(T)$$

In the following calculation, we only discuss the extra government subsidies for transition g_{trans} . Because the $R_{trans,cp}(T)$ in NYC is always negative and $-R_{trans,cp}(T) \gg \frac{1}{2}C_{trans,max}$, which means no solution for g_{total} and we cannot avoid the bus company losing fundings only with the extra government subsidies for transition g_{trans} .

Example 8: Model Development, Problem B Team #13795, The High School Affiliated to Renmin University of China, China



Team # 14211	Page 14 of 24
4. Risk Assessment Model for Invasiv	ve Species (RAMIS)
4.1. Determine Impact Factors for Invasive S	Species
Based on the intrinsic characteristics of invasive specie following some relevant reference [12, 27], we determine	s and the impact of the external environment, the impact factors for invasive species.
Define risk indicator of invasive specie (RIIS): $R (0 \le R$ invasive species determine RIIS as follows:	\leq 3). The indicators (that is impact factors) for
A. Distribution status of invasive specie: P_1 .	
B. Potential harmfulness: P_2 .	
a. Potential economic harmfulness: P_{21} .	
b. Biological media of pests spreading: P_{22} .	
c. Government attention: P_{23} .	
C. Economic impact: P_3 .	
a. Numbers of economic crop types: P_{31} .	
b. Planting area of economic crops: P_{32} .	
c. Worth of economic crops: P_{32} .	
D. Possibility of spreading: P_4 .	
a. Accessibility of invasive specie: P_{41} .	
b. Survival rate of pests during transportation: <i>H</i>	P ₄₂ .
c. Adaptability of external environments: P_{43} .	
d. Adaptability of domestic environments: P_{44}	
e. The ability of spreading: P_{45} .	
E. The difficulty of risk management: P_5	
a. The difficulty of identification: P_{51} .	
b. The difficulty of pest control treatment: $P_{\rm 52}.$	
c. The difficulty of removing invasive specie: $P_{\rm s}$	53.
Here, $0 \leq P_i \leq 3$ is the one-level indicator indicator and	$0 \le P_{ij} \le 3$ is the two-level indicator.

Example 9: Model Development, Problem A Team #14211, Shanghai World Foreign Language Academy, China



Team # 14211

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We assess whether a species qualifies as an invasive species through the examination of its risk assessment. The indicator evaluation criteria for risk assessment are as follows:

Table 4.1. Indicator evaluation criteria for risk assessment

Indicator	Value	Evaluation criteria
А	P_1	No the specie in local originally, $P_1 = 3$; Domestic distribution area accounts for $0\% \sim 20\%$, $P_1 = 2$; Account for $20\% \sim 50\%$, $P_1 = 1$; Greater than 50%, $P_1 = 0$
B.a	P_{21}	According to predictions, the resulting yield loss is over 20%, and/or the quality of crop products is severely reduced, $P_{21} = 3$; The yield loss is between 20% and 5%, and/or there is a significant quality loss, $P_{21} = 2$; The yield loss is between 5% and 1%, and/or there is a small quality loss, $P_{21} = 1$; The yield loss is less than 1% and has no impact on quality, $P_{21} = 0$. (If it is difficult to evaluate the loss of yield/quality, indirect evaluation can be considered based on the degree of harm caused by harmful organisms.)
B.b	P_{22}	Convey more than 3 kinds of quarantine pests, $P_{22} = 3$; convey 2 kinds, $P_{22} = 2$; convey 1 species, $P_{22} = 1$; does not transmit any quarantine pests, $P_{22} = 0$.
B.c	P_{23}	If more than 20 countries list a pest as a quarantine object, $P_{23} = 3$; for 19 to 10 pests, $P_{23} = 2$; for 9 to 1 pest, $P_{23} = 1$; for none, $P_{23} = 0$.
C.a	P_{31}	There are more than 10 species of cultivated hosts affected, $P_{31} = 3$; 9 ~ 5 species, $P_{31} = 2$; 4 ~ 1 species, $P_{31} = 1$; none, $P_{31} = 0$.
C.b	P_{32}	The total area of damaged cultivated hosts reaches more than 3.5 million hm ² , $P_{32} = 3$; 3.50 to 1.5 million hm ² , $P_{32} = 2$; less than 1.5 million hm ² , $P_{32} = 1$; none, $P_{32} = 0$.
C.c	P_{33}	According to its application value, export earnings, etc., experts will judge and grade it, $P_{33} = 3, 2, 1, 0$.
D.a	P_{41}	If pests are often intercepted, $P_{41} = 3$; if they are occasionally inter- cepted, $P_{41} = 2$; if they are never intercepted or only intercepted a few times in history, $P_{41} = 1$. Due to existing inspection technology, this item does not have a "0" level.
D.b	P_{42}	If the survival rate of pests during transportation is above 40%, $P_{42} = 3$; between 40% and 10%, $P_{42} = 2$; between 10% and 0, $P_{42} = 1$; the survival rate is 0, $P_{42} = 0$.

Example 9: Model Development, Problem A, continued Team #14211, Shanghai World Foreign Language Academy, China



	Table 4.1 Inc	licator evaluation criteria for risk assessment (Continued)
Indicator	Value	Evaluation criteria
D.c	P_{43}	Distributed in more than 50% of the countries in the world, $P_{43} = 3$; between 50% and 25%, $P_{43} = 2$; between 25% and 0, $P_{43} = 1$; 0, $P_{43} = 0$.
D.d	P_{44}	If more than 50% of the domestic areas are suitable for breeding, $P_{44} = 3$; between 50% and 25%, $P_{44} = 2$; between 25% and 0, $P_{44} = 1$; and the suitable range is 0, $P_{44} = 0$.
D.e	P_{51}	For airborne pests, $P_{45} = 3$; for pests transmitted by highly mobile vectors, $P_{45} = 2$; for soil-borne pests and pests with weak transmission capabilities, $P_{45} = 1$. There is no level 0 for this item.
E.a	P_{51}	The reliability of the existing inspection and identification methods is very low and takes a long time, $P_{51} = 3$; the inspection and identification methods are very reliable, simple and fast, $P_{51} = 0$; somewhere in between, $P_{51} = 2, 1$.
E.b	P_{52}	Existing pest control treatment methods are almost completely unable to kill pests, $P_{52} = 3$; pest elimination rate is below 50%, $P_{52} = 2$; pest elimination rate is between 50% and 100%, $P_{52} = 1$; pest elimination rate is 100%, $P_{52} = 0$.
E.c	P_{53}	If the field control effect is poor, the cost is high, and the difficulty is high, $P_{53} = 3$; if the field control effect is significant, the cost is low and simple, $P_{53} = 0$; if it is in between, $P_{53} = 2$, 1.

Example 9: Model Development, Problem A, Continued Team #14211, Shanghai World Foreign Language Academy, China



Team # 14087

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The table shown above shows the TOPSIS result for New York cities. We use TOPSIS model to rank the importance of each bus route and determine which bus routes should be transit into electric bus as the priority. The table above listed the top 10 bus route that should be transit into electric bus as the priority. This can be cause by high ridership, low elevation, or short distance.

6.7 Result Analysis for Charging Station 6.7.1 Seattle Charging Station

By using the Electric Bus to Charging Station ratio, we calculate that Seattle requires a total of 8 depot charging station. To see where is the best place to place the charging stations. We first find out the coordinate of the start and end station of each bus route in Seattle. Then by using k mean clustering model, we cluster all of the bus terminal coordinate into 8 cluster. The 8 cluster is shown below in the table and represent the 8 most convenient charging station location for all bus in Seattle.

Seattle	Latitude	Longitude
Charging Station 1	47.583345	-122.331067
Charging Station 2	47.668246	-122.279981
Charging Station 3	47.672441	-122.326911
Charging Station 4	47.600383	-122.329658
Charging Station 5	47.564496	-122.324989
Charging Station 6	47.465409	-122.280121
Charging Station 7	47.624046	-122.314998
Charging Station 8	47.677001	-122.346885

Table 9: 8 Location for Charging Station in Seattle

The eight coordinate shown above represent the recommended charging station position in Seattle.

6.7.2 San Francisco Charging Station

Through the bus charging station to bus number ratio, we calculate that San Francisco use needs 6 charging station. Due to the limitation of length, we only present the location of 3 charging stations that will be build in first 5 years. One notable result error is the charging station 2. The Charging station 2 is outside of San Francisco because it is accounting the special bus route 25 Treasure Island. Because the Charging Station 2 is only accounting 25 Treasure Island, We plan to directly build the station on the island.

San Francisco	Latitude	Longitude
Charging Station 1	37.712367	-122.136049
Charging Station 2	47.668246	-122.279981
Charging Station 3	37.750362	-122.48414

Table 10: 3 Location for Charging Station in San Francisco

6.7.3 New York City Charging Station

New York City has the largest size and the most number of bus; thus it has the most complicated public transport system needs to be convert. To avoid pages of different color pictures and data, we present the location of charging station in New York for the first two year. Overall, there are 31 charging station in New York after 10 years. In the First two year, there will be 6 charging station built. The following locations are where the charging station is.

Some errors with the location of the Charging Station is that the locations of Charging Station 1 and Charging Station 4 are located on the sea. To fix this error, we move the location onto the Shore to make the Charging Station plausible.

Example 10: Model Development, Problem B Team #14087, BASIS International School Guangzhou, China



Team # 14087

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New York City	Latitude	Longitude(W)			
Charging Station 1	40.429995	73.583237			
Charging Station 2	40.873	73.90875			
Charging Station 3	40.621	73.9351			
Charging Station 4	40.49395	73.552675			
Charging Station 5	40.835936	73.838569			
Charging Station 6	40.828	73.9285			
Table 11 C Leasting for Observing Obsting in New York					

Table 11: 6 Location for Charging Station in New York

6.8 Overall planning of three cities

The following maps of three cities represent the location of the routes that needs to be change and the locations of the charging stations.



Figure 9: Seattle, San Francisco, New York E-Bus Transition Plan

The color of the routes represent the different times when the route is converted to electric bus. The dark red is the route converted in first year. Red represents second year, and orange represents the third year. Yellow represents fourth year. Finally green represents the fifth year. Pink points represent the location of charging station. For Seattle and San Francisco, we presents the first five year plan while we only present the first two year plan for New York City due to its large number of buses. A complete plan will be attach to Appendix.

Example 10: Model Development, Problem B, Continued Team #14087, BASIS International School Guangzhou, China



Team #13753 Page 11 of 25 3.5 Sensitivity Analysis on Parameters: Impacts of Climate Condition In the context of the impact of climatic conditions on dandelion growth, we assume that climatic conditions exert influence on the diffusion process of dandelions by a ffecting parameters within the model. For instance, in dry and windy weather, dandelion flight speed increases, lowering the probability of descent. Conversely, in humid and rainy conditions, dandelion flight speed decreases, resulting in a smaller diffusion range [15]. Furthermore, we categorize these factors into parameters affecting the diffusion process and those influencing the growth process, as outlined in Table 5: Process Parameter Impact Factor Temperature, Humidity, Wind Velocity r Growth KTemperature, Humidity Wind Velocity y_g, σ_y Dispersal Wind Direction w_x, w_y Table 5 Impacts of Parameters Specifically, parameters influencing the growth process and dispersal process encompass: Temperature: Temperature affects dandelion processes such as photosynthesis, transpiration, and seed germination, consequently impacting the growth rate r and maximum quantity K of dandelions. • Precipitation: An increase in precipitation contributes to enhanced photosynthesis in plants and facilitates the absorption of nutrients by the soil. However, excessively high precipitation may adversely affect the root growth of dandelions. Therefore, precipitation also influences the growth rate r and maximum quantity K of dandelions. • Wind Velocity: An increase in wind speed results in an increased range of dandelion seed dispersal. Furthermore, competition between plants decreases with a greater dispersal range, which impacts the growth rate r of dandelions. Wind Direction: Wind components in the x and y directions is denoted as w_x and w_y respectively. Additionally, the wind direction changes with the seasons. Our initial analysis involves a uniform wind direction. Later, we will introduce wind speed biased towards a specific direction, thereby altering the shape of the distribution.

Example 11: Sensitivity Analysis, Problem A Team #13753, Beijing Keystone Academy, China



Team #13753 Page 12 of 25 3.5.1 Temperature With the increase in the average temperature over the 12-month period, the total dispersal area and the overall quantity of dandelions undergoes a process of initial growth followed by a subsequent decline. The comparison simulation locates with a winter temperature of 8°C, the rise in temperature allows dandelions to propagate and grow even during winter, thereby enhancing their dispersal area and total quantity. Distance from 20°C for Each Increr 0.40 0.403 0.40 0.40 0.4 0.39

Figure 8 Effects of Temperature

As illustrated in Figure 8, when the average temperature throughout the year reaches around 20° C (+ 12° C), the dispersal area and total quantity almost simultaneously reach their maximum.

Beyond 20° C, further average temperature increases lead to excessively high summer temperatures, which, in turn, inhibit the dispersal and growth of dandelions. The right portion of Figure 8 further substantiates this point. When the temperature exceeds the optimal growth temperature of 20° C, the deviation of the median temperature over 12 months from the ideal optimal growth temperature results in a decrease in the growth rate and a reduction in the number of dandelions.

3.5.2 Humidity

0 30

10

With a $0 \sim 20\%$ increase in average humidity over the 12 months, the total quantity of dandelions experiences an initial decrease followed by an increase.

The ideal humidity for dandelions is approximately around 70% [10]. In our simulated experimental scenario, the humidity from June to October closely approximates the optimal humidity. Therefore, an increase in average humidity leads to excessively high humidity in the summer, inhibiting the growth and dispersal of dandelions during that season. However, further increases in average humidity promote growth during the autumn and winter seasons and enhance the total population over 12 months as is shown in Figure 9.

> Example 11: Sensitivity Analysis, Problem A, Continued Team #13753, Beijing Keystone Academy, China



Figure 9 Effects of humidity

The bar chart on the right side of Figure 9 provides additional clarification. In this chart, the closer the median humidity over the 12 months is to the optimal growth humidity (70%), the higher the growth rate and the greater the total population. After a humidity increase of more than +10%, the median humidity's proximity to the optimal humidity decreases. More months approach the ideal humidity, resulting in higher growth rates and consequently, a larger population.

3.5.3 Wind Velocity and Directions

Larger wind speed results in dandelion dispersal over longer distances, leading to a larger dispersal area. Wind speed increases the wind speed term in Equation 14, thereby generating a flatter distribution function and allowing dandelions to spread even farther.



Figure 10 Effects of Wind Velocity

As dandelions disperse over greater distances, the competition pressure between individuals of the same species decreases, facilitating faster growth and leading to a larger population after 12 months. In our simulation, we assume that the plot of land is initially devoid of any dandelions. Therefore, a higher wind speed increases the probability of dandelion dispersal to new, unoccupied areas, reducing population competition and resulting in a higher growth rate, as indicated by the $(1 - \frac{N}{K})$ term in Equation 2.

Example 11: Sensitivity Analysis, Problem A, Continued Team #13753, Beijing Keystone Academy, China





Figure 11 Effects of Wind Directions.

With the seasonal variation in wind speed, the seed distribution also exhibits differences. Figure 11 illustrates the dispersion of seeds in different seasons and wind directions, ranging from a uniformly distributed Gaussian pattern to a skewed inverse Gaussian distribution when there is a bias in the wind direction.

Example 11: Sensitivity Analysis, Problem A, Continued Team #13753, Beijing Keystone Academy, China



14323: 15

2.8.1 Sensitivity Analysis

We incorporated sensitivity analysis to identify how variations in input values impact our model's results. We varied each variable and parameter by 10% in order to determine the significance of each variable in equation output, *T*. This can be seen in **Figure 2.8.1.1**, below.

Variable/Parameter	Definition	Effect on Output
Ι	Cost Covered by Governmental Incentives	+4.17%
В	Number of Buses Transitioned	+10%
С	Purchasing Cost of Electric Bus	-62.57%
D_P	Price of Diesel per Mile	44.96%
E_P	Price of Electricity per Mile	-24.33%
F_{E}	Maintenance cost per Mile (Electric)	-29.15%
F_D	Maintenance cost per Mile (Diesel)	+81.1%
M	Lifetime Distance Traveled Per Bus	+72.6%
R	Residual Value	+11.04%

Figure 2.8.1.1: Sensitivity Analysis for Financial Considerations

The sensitivity analysis reveals that variations in the Purchasing Cost of each Electric Bus, Price of Diesel per Mile, Price of Electricity per Mile, Maintenance cost per Mile (Electric), Maintenance cost per Mile (Diesel), and Lifetime Distance Traveled Per Bus had outsized (> 20%) impacts on our results. Thus, we determined these factors to be both the most *influential* in our model and therefore the most important to model in future work to accurately predict the total cost of transitioning to an all-electric bus fleet.



Team # 14555 Page 20 of 22 **11** Strengths and Weaknesses Strength 1. Comprehensive consideration. This model takes a comprehensive approach to evaluating the feasibility of electric buses as a viable mode of transportation. It not only considers the number of electric buses but also takes into account critical factors such as supporting infrastructure, charging facilities, and the required workforce. By incorporating these elements, the model provides a more nuanced and realistic assessment of the scalability of electric buses, offering valuable insights into the future of transportation. The model goes beyond a narrow focus on emission reduction and addresses a spectrum of ecological consequences such as air pollution, particulate matter pollution, and noise pollution. This holistic approach ensures a more comprehensive understanding of the overall impact of transitioning from conventional fuel-powered buses to electric buses. Strength 2. Highly adaptable and can be widely applied. λ allows users to freely define adjustments to make the model more flexible and able to adapt to different data and requirements. More precisely, the parameter λ can be fine-tuned to strike a balance between enhancing ecological benefits and minimizing financial costs. When $\lambda = 0.5$, it indicates equal importance assigned to both ecological benefits and financial costs. For instance, with $\lambda =$ 0.8 > 0.5, financial costs take precedence, while with $\lambda = 0.2 < 0.5$, ecological consequences become more prioritized. Strength 3. User-friendly and highly effective. The model's transparent calculations make it easy to understand and access. By highlighting the relationships among different variables and their environmental consequences, the model provides clarity in demonstrating the impact of diesel and electric buses. It goes beyond a narrow focus on emission reduction, addressing a range of ecological consequences like air pollution, particulate matter pollution, and noise pollution. This holistic approach ensures a more comprehensive understanding of the overall impact of transitioning from conventional fuel-powered buses to electric buses. Weakness 1. Timeliness of data. It is important to emphasize that while the total annual mileage data for electric buses is a crucial requirement, the data used in the model should not only be comprehensive but also up-to-date. With the rapid development of electric bus technology and infrastructure, the use of current data is essential to improve the accuracy of predictions and forecasts. Regular updates of the dataset will ensure that the model remains relevant and reliable in dynamic and changing scenarios. Weakness 2. Disadvantages of Monte Carlo simulation. Monte Carlo simulation is a computational method that has been recognized as a feasible approach, despite its intensive nature. However, there are potential challenges associated with its practical implementation that need to be thoroughly explored. To achieve more realistic and real-life applications, it is necessary to delve into the practical aspects of implementing Monte Carlo simulations, such as optimizing computing resources and managing large-scale calculations. There are certain challenges posed by the broader feasibility and potential limitations of the model, and it is necessary to refine the model in more detail.

Example 13. Strengths and Limitations, Problem B Team 14555, Shanghai Linstitute School, China



Team # 14140

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4 Conclusion

In Section 2, we provided our solution to problem 1. First, we created a simple model, simulating the spread of dandelions using the negative exponential distribution. However, this does not accurately model the path a dandelion seed takes during its dispersal, nor does it take into account any external factors. Thus, we then utilized the lévy flight model, which uses the lévy distribution curve instead. This model more accurately presents the path a dandelion seed takes during its dispersal. This model is still insufficient, though, as it still does not take into account external factors, nor does it model the spread of dandelions overtime. Therefore, we constructed a death rate that is influenced by factors such as temperature, precipitation or population density using the logistic growth model. With this death rate, we were then finally able to properly estimate the spread of dandelions overtime, using a dynamic model where the death rates in each condition are updated daily. Using this model, we were able to model the spread of dandelions over 1, 2, 3, 6, and 12 months in tropical, dry temperate and continental climates.

In section 3, we provided our solution to problem 2. We first split the determination of the impact factor into different components due to there being many variables that should be considered. Next, since different variables have various extents in determining the final impact factor, we decided to assign weights to each of them using the Analytical Hierarchy Process (AHP) and the Entropy Weight Method (EWM) combined to obtain an objective result. It was decided that there would be three main determinants of an invasive plant's impact factor: its economical, ecological impacts, and its dispersal ability. These were then further split into smaller criteria that would affect the three determinants. We used the weights obtained using the AHP and EWM models to calculate individual impact scores for each of the determinants. In the end, we combined these scores, yielding a final impact factor ranging from 0 to 5. On top of this, we determined that if a plant has an impact factor that is greater or equal to 3.5, it is considered invasive. Our results show that dandelions and Miscanthus sinensis are not considered invasive but have moderate effects on the economy and environment, whereas the Purple Loosestrife is considered invasive in all four climatic conditions.

Even though our models are rather basic compared to those constructed in real world scenarios, there are still interesting phenomena worth noting. One major characteristic we observed is that even though the death rate in a tropical climate is lower compared to a continental climate, after 12 months there were more surviving seeds in the continental climate. This lead us to the conclusion that perhaps when the death rate of two invasive plants are not significantly different, the distance of dispersal is a much more decisive factor in determining their eventual population.

To conclude, invasive species can have significant impacts on the environment and our societies. During our modeling process, we have witnessed the destructive capabilities of certain invasive plants. Therefore, the issue of invasive organisms should not, and cannot be underestimated.



Team 14789

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8. Letter to Bus Company

Dear CATS,

We have been tasked with assessing the ecological and financial effects of transitioning to a fully electric bus fleet. As CATS has been a leader in striving towards a more sustainable public transportation system, we want to push that forward with our plan. We have considered both the financial costs and ecological effects of this transition, and have determined the most optimal way for CATS to transition to a fully electric bus fleet in a 10-year period.

To give CATS more flexibility in their plan, we propose two different 10-year plans, one that is more financially optimal and one that is ecologically optimal. In order to replace the 286 diesel and hybrid buses that CATS have with electric buses, we present the following options below:

	# of years to break even	# of buses and chargers to buy per year (year 1, year 2 year 10)	Maximum money spent in one year	Emissions of CO2 saved in 10 years (tons)
Financially optimal	18	(11,15,19,22,26,30,35,39,42,46)	\$36,278,955	162,759
Ecologically optimal	19	(46,42,39,35,30,26,22,19,15,11)	\$58,923,172	244,139

In the financially optimal plan, we propose that CATS should buy an increasing number of buses every year. Since bus prices are decreasing, money is saved by buying more buses later. CATS would only have to spend at most 18 million dollars a year since 50% of transition costs are covered through external funding. This plan would save 163 thousand tons of CO2 in 10 years.

In the ecologically optimal plan, CATS should buy a decreasing number of buses every year. CATS would have to spend at most 29 million dollars a year, but his plan would save 244 thousand tons of CO2 in 10 years.

In the year 2022, CATS spent \$188,940,009 on operating expenses alone. Due to this, we believe that CATS will have sufficient funding to implement either of the plans proposed.

We hope that CATS will eventually implement one of our plans, to provide a more sustainable future for the city of Charlotte.





Example 16. Infographic, Problem A Team 13719, Nanjing Foreign Language School, China



Al Use Report

In this document, we gathered some of the questions that we asked AI. While this list does not include every single one of our queries, it is a good representation of how we used AI throughout our modelling process.

OpenAl GhatGPT3.5, September 2021 version

Query 1: Turn this into python matrix: Plant a b c d e f g h i j Dandelion 1500 10 10 0.5 0.3 1 0 0 1 2 Japanese Knotweed 50,000 15 18 1.0 2.5 2 2 1 0 3 Pampas grass 100,000 10 14 3.5 2.0 2 1 2 0 4

Output:

To represent the given data as a matrix in Python, you can use the NumPy library. Here's an example of how you can create a matrix with the provided values:

```
import numpy as np
# Define the data as a 2D list
data = [
    [1500, 10, 10, 0.5, 0.3, 1, 0, 0, 1, 2],
    [50000, 15, 18, 1.0, 2.5, 2, 2, 1, 0, 3],
    [100000, 10, 14, 3.5, 2.0, 2, 1, 2, 0, 4]
]
# Create the matrix using NumPy
matrix = np.array(data)
# Print the matrix
print(matrix)
```

In this example, the data is represented as a 2D list, where each inner list corresponds to a row of the matrix. The <code>np.array</code> function from NumPy is then used to convert the data into a matrix.

Running this code will display the matrix:



```
[[1.5e+03 1.0e+01 1.0e+01 5.0e-01 3.0e-01 1.0e+00 0.0e+00 0.0e+00 1.0e+00 2.0e+0
[5.0e+04 1.5e+01 1.8e+01 1.0e+00 2.5e+00 2.0e+00 2.0e+00 1.0e+00 0.0e+00 3.0e+0
[1.0e+05 1.0e+01 1.4e+01 3.5e+00 2.0e+00 2.0e+00 1.0e+00 2.0e+00 0.0e+00 4.0e+0
```

Each row of the matrix represents a different plant, and the columns represent the attributes a, b, c, d, e, f, g, h, i, j for each plant.

Query 2:

Turn this into python matrix: Min Max Reproductive rate. 1 1,000,000 Spread distance. 0 150 Lifespan. 1 20 Length of root. 0 7 Height of plant. 0 5 Water. 0 2 Sunlight. 0 2 Resistance to herbivores 0 2 Benefits 0 1 Economic costs 0 4

Output:

To represent the given data as a matrix in Python, you can use the NumPy library. Here's an example of how you can create a matrix with the provided minimum and maximum values:

```
import numpy as np
# Define the data as a list of tuples
data = [
    (1, 1000000), # Reproductive rate
    (0, 150), # Spread distance
    (1, 20),
                  # Lifespan
                 # Length of root
    (0, 7),
                # Length of Foot
# Height of plant
# Water
# Sunlight
# Resistance to herbivores
    (0, 5),
    (0, 2),
    (0, 2),
    (0, 2),
    (0, 1),
                   # Benefits
    (0, 4)
                    # Economic costs
]
```



Create the matrix using NumPy
matrix = np.array(data)

Print the matrix
print(matrix)

In this example, the data is represented as a list of tuples, where each tuple contains the minimum and maximum values for a specific attribute. The np.array function from NumPy is then used to convert the data into a matrix.

Running this code will display the matrix:

[[1	1000000]
[0	150]
[1	20]
[0	7]
[0	5]
[0	2]
[0	2]
[0	2]
[0	1]
[0	4]]

Each row of the matrix represents an attribute, and the first column corresponds to the minimum value, while the second column corresponds to the maximum value for that attribute.

Query 3:

write a program that draws a pie chart base on a vector, the value in the vector adds up to one

Output:

Certainly! Here's a Python program that uses matplotlib to draw a pie chart based on a vector where the values add up to one:

```
import matplotlib.pyplot as plt
```

```
def draw_pie_chart(vector):
    # Create a figure and axis
    fig, ax = plt.subplots()
    # Draw the pie chart
    ax.pie(vector, labels=vector, autopct='%1.1f%%')
```



Set the aspect ratio to be equal so that the pie is drawn as a circle
ax.axis('equal')

```
# Display the pie chart
plt.show()
```

```
# Example usage
vector = [0.3, 0.2, 0.1, 0.4]
draw_pie_chart(vector)
```

In this program, the draw_pie_chart function takes a vector as input and uses matplotlib's pie function to draw the pie chart. The labels argument is set to the vector itself, and the autopct argument formats the percentage labels on the chart.

You can customize the vector variable with your own vector data, as long as the values add up to one. The program will draw a pie chart representing the proportions of the values in the vector.

Query 4:

turn this list into python list: Reproductive rate. Spread distance. Lifespan. Length of root. Height of plant. Water. Sunlight. Resistance to herbivores Benefits Economic costs

Output:

To turn the given list into a Python list, you can simply enclose the items within square brackets. Here's the modified list:

```
attributes = [
    "Reproductive rate",
    "Spread distance",
    "Lifespan",
    "Length of root",
    "Height of plant",
    "Water",
    "Sunlight",
    "Resistance to herbivores",
    "Benefits",
    "Economic costs"
]
```

Now, the list attributes contains the provided attributes as individual string elements.

Query 5:

Is there any established way to quantify how invasive a species is?



Output:

Yes, there are established methods to quantify how invasive a species is. These methods typically consider various factors to assess the impact and potential spread of a species in an ecosystem. Some common metrics used to quantify invasiveness include:

- Ecological Impact: This metric evaluates the ecological disruption caused by an invasive species. It considers factors such as changes in species composition, alteration of ecosystem processes, and displacement of native species.
- Spread and Rate of Spread: This metric assesses the speed and extent at which the invasive species can colonize new areas or expand its range. It considers factors such as reproductive rates, dispersal mechanisms, and ability to establish populations in different habitats.
- 3. Competitive Ability: This metric measures the invasive species' ability to outcompete native species for resources such as food, water, and habitat. It considers factors like growth rate, resource use efficiency, and tolerance to environmental conditions.
- 4. Economic Impact: This metric quantifies the economic costs associated with the presence of an invasive species. It includes factors like damage to crops, impacts on industries (e.g., fishing, forestry), and costs of management and control efforts.
- 5. Genetic Impact: This metric examines the potential for genetic hybridization and introgression between invasive and native species. It considers factors such as genetic compatibility, reproductive barriers, and potential for genetic assimilation.

These metrics, along with others, are used in combination to assess the invasiveness of a species. Different scoring systems and indices have been developed to assign a numerical value or rank to the invasiveness level of a species, allowing for comparisons between different invasive species.

Query 6:

modify the code so that the caption is bigger def ScatterPlot(coords, valmin, valmax, i):

Extract x and y values from the coordinates x_values = [coord[0] for coord in coords] y_values = [coord[1] for coord in coords] c_values = [coord[4] for coord in coords]



```
Create the scatter plot
scatter = plt.scatter(x_values, y_values, s = 0.5, c = c_values, cmap='viridis', vmin=valmin,
vmax=valmax)
```

Set the x and y axis limits plt.xlim(0, 100) plt.ylim(0, 100)

Set labels for x and y axes plt.xlabel('X') plt.ylabel('Y')

Set the title of the plot plt.title('Temperate Dandelion Spread Scatter Plot (Day %s)' % i)

plt.colorbar(scatter, label='Age (days)', orientation='vertical', pad=0.05)

```
Display the plot
plt.savefig(dir + 'temperate' + str(i) + '.pdf')
plt.show()
```

Output:

To make the caption (title) of the plot bigger, you can modify the code by adjusting the font size using the plt.title function. Here's the modified code:

```
def ScatterPlot(coords, valmin, valmax, i):
    # Extract x and y values from the coordinates
    x_values = [coord[0] for coord in coords]
    y_values = [coord[1] for coord in coords]
    c_values = [coord[4] for coord in coords]

    # Create the scatter plot
    scatter = plt.scatter(x_values, y_values, s = 0.5, c = c_values, cmap='virid
    # Set the x and y axis limits
    plt.xlim(0, 100)
    plt.ylim(0, 100)

    # Set labels for x and y axes
    plt.xlabel('X')
    plt.ylabel('Y')

    # Set the title of the plot with increased font size
    plt.title('Temperate Dandelion Spread Scatter Plot (Day %s)' % i, fontsize=1
```



1. OpenAI Ch	atGPT (Nov 6, 2023 version, ChatGPT-4)
Query 1	
Please conve	ert the following information into a BibLaTeX @book entry.
Book Title	
Multiple Att	ribute Decision Making
Book Subtitl	e Analisations A Chata a Catha Ant Commun
Methods and	Applications A State-of-the-Art Survey
Ching-Lai Hw	yang, Kwangsun Yoon
Series Title	
Lecture Note	s in Economics and Mathematical Systems
DOI https://doi	ama /10 1007/079 2 642 49219 0
Publisher	org/10.1007/978-3-642-48318-9
Springer Ber	lin, Heidelberg
eBook Packag	les
Springer Boo	k Archive
Copyright In	formation
Softcover IS	BN
978-3-540-10	1558-9
Published: 0	1 February 1981
eBook ISBN	
978-3-642-48	318-9 16 December 2012
Series ISSN	o December 2012
0075-8442	
Series E-ISS	N
2196-9957	
Edition Numb	er
I Number of Pa	200
XI, 269	ges
Topics	
Operations R	esearch/Decision Theory, Computational Intelligence,
Economic The	ory/Quantitative Economics/Mathematical Methods
Output	
@book{HwangY	'oon1981.
author	= {Ching-Lai Hwang and Kwangsun Yoon},
title	<pre>= {Multiple Attribute Decision Making},</pre>
subtitle	<pre>= {Methods and Applications A State-of-the-Art Survey},</pre>
publisher	<pre>= {Springer-Verlag Berlin Heidelberg},</pre>
year	$= \{1981\},$ $= \{10, 1007/078 - 3 - 642 - 48318 - 0\}$
ishn	$= \{978 - 3 - 540 - 10558 - 9\},$

Example 18. AI Usage Report, Problem B Team 13921, Shanghai Linstitute School, China





Example 18. AI Usage Report, Problem B, Continued Team 13921, Shanghai Linstitute School, China



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Here's	a basic step-by-step process of how AHP works:
Moo	del the Problem as a Hierarchy: This involves breaking
dow	wn the problem into its constituent parts, forming a goal,
cr:	iteria, subcriteria, and alternatives.
Est	tablish Priorities: Assign a numerical weight to each element
of	the hierarchy. This is often done through pairwise compari-
son	n matrices, where each element is compared directly to every
oth	her element in terms of which one more strongly satisfies
the	e criterion.
Con	nsistency Ratio: Check the consistency of the judgments.
Hur	mans aren't perfectly consistent, so Saaty introduced a
mea	asure called the Consistency Ratio (CR) to capture this.
If	CR<0.10, the ratio indicates a reasonable level of consis-
ter	ncy in judgment.
Syn	nthesize the Results: The weights of the elements are com-
bin	ned from the bottom of the hierarchy upwards (synthesized).
The	e result is a priority vector or ranking of the decision
alt	ternatives.
Rev	view and Revise the Model: As with any decision-making tool,
the	e results should be checked for robustness and sensitivity
to	changes in the judgments.
AHP has	s been applied in various fields like business, education,
govern	ment, and healthcare, for making decisions that involve
multip	le criteria or alternatives.
4. Open	AI ChatGPT (Nov 6, 2023 version, ChatGPT-3.5)
Query 1	
What is	s your version?
I am ba there l trainin My resp	ased on the GPT-3.5 architecture, developed by OpenAI. If have been updates or newer versions released since my last ng cut-off in January 2022, I would not be aware of them. ponses are generated based on the knowledge available up to ate.

Example 18. AI Usage Report, Problem B, Continued Team 13921, Shanghai Linstitute School, China



5. Microsoft New Bing, Creative (used on Nov 13, 2023) Query 1 Give me a clear, professional and creative name for the mathematical model described below. The model breaks the problem down into multiple parallel ways a bus might cause ecological impacts, namely Production and Retirement, Employment, and External Impacts; each of them can impact one or more fields of impact. A TOPSIS model is used to assess resource depletion in Production and Retirement. A model based on physical principles is used to assess pollution and carbon emissions in Employment. People's ecological consciousness and its impact on carbon emissions is assessed in External Impacts. An improved version of the AHP model is utilized to combine the factors for short-term and long-term influences, respectively. Output One possible name for the mathematical model described is PEECE: Parallel Ecological Evaluation of City Buses. This name reflects the main features of the model, such as breaking the problem down into parallel ways, evaluating the ecological impacts of city buses, and using different methods for different aspects of the problem. The name is also clear, professional, and creative, as it is easy to pronounce, understand, and remember. 6. Microsoft New Bing, Precise (used on Nov 11, 2023) Query 1 How many subway stations were constructed in New York from 2015 to 2023? Output Sorry! That's on me, I can't give a response to that right now. What else can I help you with?

Example 18. AI Usage Report, Problem B, Continued Team 13921, Shanghai Linstitute School, China