1991: The Water Tank Problem

Some state water-right agencies require from communities data on the rate of water use, in gallons per hour, and the total amount of water used each day. Many communities do not have equipment to measure the flow of water in or out of the municipal tank. Instead, they can measure only the *level* of water in the tank, within 0.5% accuracy, every hour. More importantly, whenever the level in the tank drops below some minimum level L, a pump fills the tank up to the maximum level, H; however, there is no measurement of the pump flow, either. Thus, one cannot readily relate the level in the tank to the amount of water used while the pump is working, which occurs once or twice per day, for a couple of hours each time.

Estimate the flow out of the tank $f(t)$ at all times, even when the pump is working, and estimate the total amount of water used during the day. **Table 1** gives real data, from an actual small town, for one day.

The table gives the time, in seconds, since the first measurement, and the level of water in the tank, in hundredths of a foot. For example, after 3316 seconds, the depth of water in the tank reached 31.10 feet. The tank is a vertical circular cylinder, with a height of 40 feet and a diameter of 57 feet. Usually, the pump starts filling the tank when the level drops to about 27.00 feet, and the pump stops when the level rises back to about 35.50 feet.

Table 1.

Water-tank levels over a single day for a small town. Time is in seconds and level is in 0.01 ft.

Time	Level	Time	Level	Time	Level
0	3175	35932	pump on	68535	2842
3316	3110	39332	pump on	71854	2767
6635	3054	39435	3550	75021	2697
10619	2994	43318	3445	79254	pump on
13937	2947	46636	3350	82649	pump on
17921	2892	49953	3260	85968	3475
21240	2850	53936	3167	89953	3397
25223	2797	57254	3087	93270	3340
28543	2752	60574	3012		
32284	2697	64554	2927		

Comments by the Contest Director

The problem was contributed by Yves Nievergelt (Mathematics Dept., Eastern Washington University, Cheney, WA). It is based on data from a consulting problem for Union, a town of 11,500 in northeastern Oregon. The Outstanding papers inspired immediate applications at the consulting firm, Equipment Technology and Design.

The Outstanding papers were by teams from Hiram College, Ripon College, and University of Alaska Fairbanks. Their papers, together with commentaries, were published in *The UMAP Journal* 12 (3) (1991): 201–241.