1994: The Concrete Slab Problem

The U.S. Dept. of Housing and Urban Development (HUD) is considering constructing dwellings of various sizes, ranging from individual houses to large apartment complexes. A principal concern is to minimize recurring costs to occupants, especially the costs of heating and cooling. The region in which the construction is to take place is temperate, with a moderate variation in temperature throughout the year.

With special construction techniques, HUD engineers can build dwellings that do not need to rely on convection—that is, there is no need to rely on opening doors or windows to assist in temperature variation. The dwellings will be single-story, with concrete slab floors as the only foundation. You have been hired as a consultant to analyze the temperature variation in the concrete slab floor to determine if the temperature averaged over the floor surface can be maintained within a prescribed comfort zone throughout the year. If so, what size/shape of slabs will permit this?

Part 1, Floor Temperature

Consider the temperature variation in a concrete slab given that the ambient temperature varies daily within the ranges given in **Table 1**. Assume that the high occurs at noon and the low at midnight. Determine if slabs can be designed to maintain a temperature averaged over the floor surface within the prescribed comfort zone considering radiation only. Initially, assume that the heat transfer into the dwelling is through the exposed perimeter of the slab and that the top and bottom of the slabs are insulated. Comment on the appropriateness and sensitivity of these assumptions. If you cannot find a solution that satisfies **Table 1**, can you find designs that satisfy a **Table 1** that you propose?

Table 1.
Daily variation in temperature.

Ambient Temperature		Comfort Zone	
High:	85°F	High:	76°F
Low:	60°F	Low:	65°F

Part 2, Building Temperature

Analyze the practicality of the initial assumptions and extend the analysis to temperature variation within the single-story dwelling. Can the house be kept within the comfort zone?

Part 3, Cost of Construction

Suggest a design that considers HUD's objective of reducing or eliminating heating and cooling costs, considering construction restrictions and costs.

Comments by the Contest Director

The problem is based on a heat-diffusion problem (a slab without insulation) by Murray Klamkin (Mathematics Dept., University of Alberta, Alberta, Canada).

The Outstanding paper was by a team from North Carolina School of Science and Mathematics. Their paper, together with a commentary, was published in *The UMAP Journal* 15 (3) (1994): 207–224.